Mixture Models with Missing data
Classification of Satellite Image Time Series
QUALIMADOS: Atelier Qualité des masses de données scientifiques

S. Iovleff

Laboratoire Paul Painlevé

23 Juin 2017

Sommaire

Clustering is the cluster building process

Cluster analysis

From Wikipedia, the free encyclopedia

For the supervised learning approach, see Statistical classification.

Cluster analysis or **clustering** is the task of grouping a set of objects in such a way that objects in the same group (called a **cluster**) are more similar (in some sense or another) to each other than to those in other groups (clusters). It is a main task of exploratory data mining, and a common technique for statistical data analysis, used in many fields, including machine learning, pattern recognition, image analysis, information retrieval, bioinformatics, data compression, and computer graphics.

- The term Data Clustering first appeared in 1954 (according to JSTOR) in an article dealing with anthropological data,
- Many, many existing methods (`https://en.wikipedia.org/wiki/Category:Data_clustering_algorithms`)

Clustering is the cluster building process

Cluster analysis

From Wikipedia, the free encyclopedia

For the supervised learning approach, see Statistical classification.

Cluster analysis or **clustering** is the task of grouping a set of objects in such a way that objects in the same group (called a **cluster**) are more similar (in some sense or another) to each other than to those in other groups (clusters). It is a main task of exploratory data mining, and a common technique for statistical data analysis, used in many fields, including machine learning, pattern recognition, image analysis, information retrieval, bioinformatics, data compression, and computer graphics.

- The term Data Clustering first appeared in 1954 (according to JSTOR) in an article dealing with anthropological data,
- Many, many existing methods (https://en.wikipedia.org/wiki/Category:Data_clustering_algorithms)

Clustering is the cluster building process

Cluster analysis

From Wikipedia, the free encyclopedia

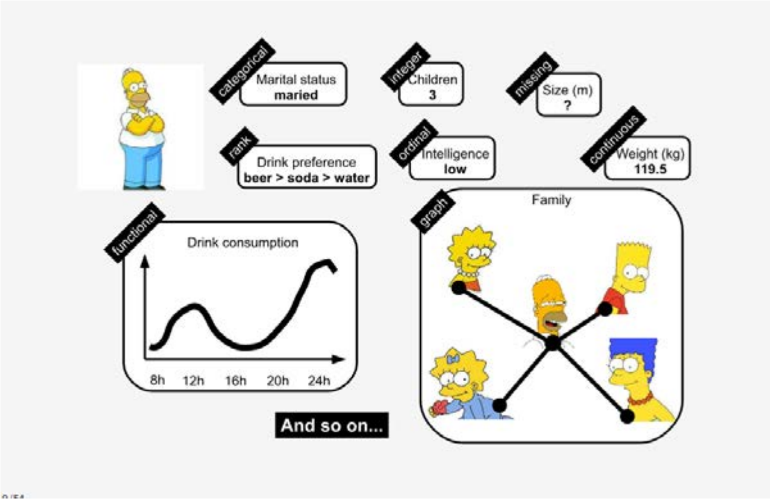
For the supervised learning approach, see Statistical classification.

Cluster analysis or **clustering** is the task of grouping a set of objects in such a way that objects in the same group (called a **cluster**) are more similar (in some sense or another) to each other than to those in other groups (clusters). It is a main task of exploratory data mining, and a common technique for statistical data analysis, used in many fields, including machine learning, pattern recognition, image analysis, information retrieval, bioinformatics, data compression, and computer graphics.

- The term Data Clustering first appeared in 1954 (according to JSTOR) in an article dealing with anthropological data,
- Many, many existing methods (`https://en.wikipedia.org/wiki/Category:Data_clustering_algorithms`)

New challenges

Need to algorithms for Big-Data and Complex Data. In particular mixed features and missing values



An example

Joint works with Christophe Biernacki (head of the Inria Modal team), Vincent Vandewalle, Komi Nagbe,...

Contract for a large lingerie store: "*Clustering cash receipts of the Customers with a loyalty card*"

- 28 variables related to products,
- 6 variables related to costumers,
- 8 variables related to stores,
- $n = 2,899,030$ receipts.

Some meaningful variables with missing values.

An example

Joint works with Christophe Biernacki (head of the Inria Modal team), Vincent Vandewalle, Komi Nagbe,...

Contract for a large lingerie store: "*Clustering cash receipts of the Customers with a loyalty card*"

- 28 variables related to products,
- 6 variables related to costumers,
- 8 variables related to stores,
- $n = 2,899,030$ receipts.

Some meaningful variables with missing values.

An example

Joint works with Christophe Biernacki (head of the Inria Modal team), Vincent Vandewalle, Komi Nagbe,...

Contract for a large lingerie store: "*Clustering cash receipts of the Customers with a loyalty card*"

- 28 variables related to products,
- 6 variables related to costumers,
- 8 variables related to stores,
- $n = 2,899,030$ receipts.

Some meaningful variables with missing values.

An example

Joint works with Christophe Biernacki (head of the Inria Modal team), Vincent Vandewalle, Komi Nagbe,...

Contract for a large lingerie store: "*Clustering cash receipts of the Customers with a loyalty card*"

- 28 variables related to products,
- 6 variables related to costumers,
- 8 variables related to stores,
- $n = 2,899,030$ receipts.

Some meaningful variables with missing values.

An example (Variables)

Variables liées aux clients

Num	Name_var	Type	Nbre Modal	Modalités	% Manquant	Discrétisé
1	CODE_CIVILITE	factor	4	MLE|MM|MME|MR	32.55	NON
2	TRANCHE_URBAINE	factor	5	+100000|20000 à 50000|-5000|50000 à 100000|5000 à 20000	33.37	NON
[redacted]	[redacted]	factor	2	NON|OUI	0	NON
[redacted]	[redacted]	factor	2	NON|OUI	0	NON
5	1ANNEE_NAISSANCE (Âge)	numeric	106	0	37.07	NON
[redacted]	[redacted]	integer	225	0	32.55	NON

Variables liées aux magasins

Num	Name_var	Type	Nbre Modal	Modalités	% Manquant	Discrétisé
1	TYPE_MAGASIN	factor	2	FRANCHISE|SUCCURSALE	0	NON
2	SECTEUR_MAGASIN	factor	10	[redacted]	[redacted]	NON
3	LOCALISATION_MAGASIN	factor	4	CENTRE CIAL AVEC HYPERMARCHE|CENTRE CIAL SANS HYPERMAR	0.41	NON
4	MUSIQUE_MAGASIN	factor	2	NON|OUI	0	NON
5	magasin_ferme	factor	2	FERME|NON FERME	0	NON
6	[redacted]	factor	4	1-5ans|6-10ans|11-15ans|plus de 15ans	0	NON
[redacted]	[redacted]	factor	28		0	NON
8	SUPERFICIE_MAGASIN	integer	53	0	0	NON

An example (Variables)

Variables liées aux Produits

Num	Name_var	Type	Nbre Moda	Modalités		% Manqua	Discrétis
1	mois_ticket	factor	12	01|02|03|04|05|06|07|08|09|10|11|12		0	0
2	jours_ticket	factor	7	dimanche[redacted]di|lundi|mardi|mercredi|samedi|vendredi		0	0
3	TYPE_LIGNE	factor	[redacted]	[redacted]		0	0
5	MONTANT_REMISE	factor	[redacted]	[redacted]|SANS REMISE		0	OUI
6	SOLDE	factor	2	NON|OUI		0	0
7	MT_REMISE_OP_COMM	factor	2	REMISE COMM|SANS REMISE COMM		0	OUI
[redacted]		factor	2	NON|OUI		0	0
9	[redacted]	factor	2	NON|OUI		0	0
10	REGLEMENT_CB	factor	2	NON|OUI		0	0
11	REGLEMENT_ES	factor	2	NON|OUI		0	0
12	REGLEMENT_CH	factor	2	NON|OUI		0	0
13	REGLEMENT_AUTRES	factor	2	NON|OUI	"REGLEMENT_KK","REGLEMENT_CB","REGLEMENT_ES","REGLEMENT_C	0	0
14	KKDO_UTILISATION	factor	2		NON|OUI	0	0
1[redacted]		factor	1		NON	0	0
16[redacted]		factor	2	NON|OUI		0	0
17[redacted]		factor	2	NON|OUI		0	0
18	[redacted]VELLE_CLIENTE	factor	2	NON|OUI		0	0
1[redacted]		factor	2	NON|OUI	[redacted]	0	0
2[redacted]		factor	2	NON|OUI	[redacted]	0	0
21	GROUPE_PRODUIT	factor	5		ACCESSOIRES MODE|BALNEA[redacted]S|LINGERIE DE JOUR|LIN	0	0
22	COLLECTION	factor	2		[redacted]	0	0
23	COLORIS_BASE	factor	13		[redacted]une|Marron|Noir|Orange|Peau|Rose|Rouge|Vert|Viol	0	0
24	COORDONNE	factor	4		[redacted]	19.02	0
25	STYLE_PORTER	factor	4		CONFOR[redacted]SEDUCTION|SEXY[redacted]	19.74	0
26	ASPECT_COLORIS	factor	3		BICOLORE|IMPRIME|UNI	2.35	0
27	CARACT_MATIERE	factor	5		AUTRES|DENTELLE|MICROFIBRE|SATIN|TULLE BRODE	0	0
4	PRIX_UNITAIRE	numeric	4811		0	0	0

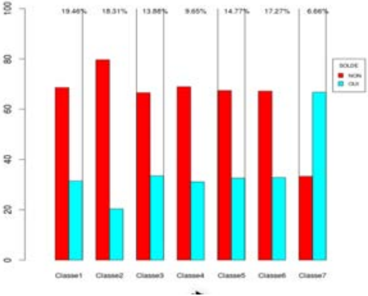
An example (Results)

	Classe1	Classe2	Classe3	Classe4	Classe5	Classe6	Classe7
Proportion	[redacted]	18.31%	[redacted]	[redacted]	14.77%	[redacted]	6.66%
Effectif	[redacted]	527499	[redacted]	[redacted]	427417	[redacted]	205443

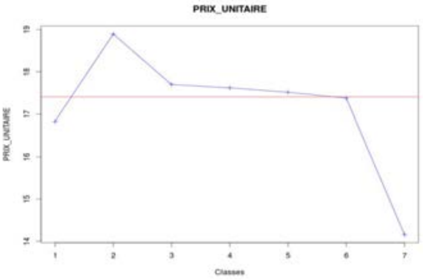
An example (Results)

	Classe1	Classe2	Classe3	Classe4	Classe5	Classe6	Classe7
Proportion	[redacted]	18.31%	[redacted]	[redacted]	14.77%	[redacted]	6.66%
Effectif	[redacted]	527499	[redacted]	[redacted]	427417	[redacted]	205443

Solde



Prix Unitaire

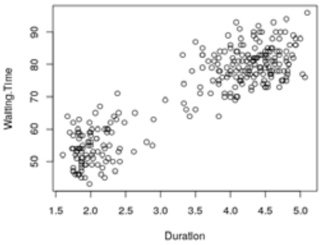


Mixture Models

Main Idea

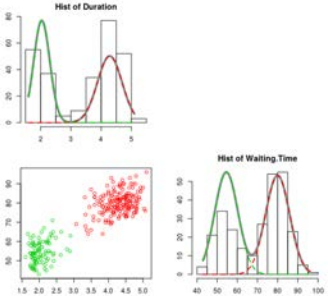
$$\mathbf{x} \text{ in cluster } k \iff \boldsymbol{x} \text{ belongs to distribution } P_k$$

$\boxed{\mathbf{x} = (\mathbf{x}_1, \ldots, \mathbf{x}_n)}$



$\overrightarrow{\text{clustering}}$

$\boxed{\hat{\mathbf{z}} = (\hat{\mathbf{z}}_1, \ldots, \hat{\mathbf{z}}_n),\ \hat{K} \text{ clusters}}$



Model Based clustering is a probabilistic approach.

R package MixAll and SaaS MixtComp

Two softwares available

- R package MixAll

```
> library(MixAll)
> data(geyser)
> ## add 10 missing values as random
> x = as.matrix(geyser); n <- nrow(x); p <- ncol(x);
> indexes <- matrix(c(round(runif(5,1,n)), round(runif(5,1,p))), ncol=2);
> x[indexes] <- NA;
> ## estimate model
> model<-clusterDiagGaussian( data=x, nbCluster=2:3, models=c( "gaussian_pk_sjk"))
> plot(model)
> missingValues(model)
  row col     value
1 133   1  2.029661
2  42   2 54.569144
3  49   2 79.970973
4 209   2 54.569144
5 213   2 54.569144
```

- SaaS software MixtComp https://massiccc.lille.inria.fr/

Hypothesis of mixture of parametric distributions

- Cluster k is modeled by a parametric distribution

$$\mathbf{x}_i | z = k \sim p(.|\alpha_k)$$

- Cluster k has probability π_k

$$z_i \sim \mathcal{M}(1, \pi_1, \ldots, \pi_K).$$

Mixture model

The model parameters are $\theta = (\pi_1, \ldots, \pi_K, \alpha_1, \ldots, \alpha_K)$ and

$$p(\mathbf{x}_i) = \sum_{k=1}^{K} \pi_k p(\mathbf{x}_i; \alpha_k)$$

EM Algorithm

Starting from an initial arbitrary parameter θ^0, the mth iteration of the EM algorithm consists of repeating the following **I**, **E** and **M** steps.

- **I step:** *Impute* by using expectation of the missing values $\mathbf{x}^m$ using $\mathbf{x}^o$, θ^{r-1}, t_{ik}^{r-1}.
- **E step:** Compute conditional probabilities $z_i = k|\mathbf{x}_i$ using current value θ^{r-1} of the parameter:

$$t_{ik}^r = t_k^r(\mathbf{x}_i|\theta^{r-1}) = \frac{p_k^{r-1} h(\mathbf{x}_i|\alpha_k^{r-1})}{\sum_{l=1}^K p_l^{r-1} h(\mathbf{x}_i|\alpha_k^{r-1})}. \tag{1}$$

- **M step:** Update ML estimate θ^r using conditional probabilities t_{ik}^r as mixing weights

$$L(\theta|\mathbf{x}_1,\ldots,\mathbf{x}_n,\mathbf{t}^r) = \sum_{i=1}^n \sum_{k=1}^K t_{ik}^r \ln\left[p_k h(\mathbf{x}_i|\alpha_k)\right],$$

- Iterate until convergence

EM Algorithm

Starting from an initial arbitrary parameter θ^0, the mth iteration of the EM algorithm consists of repeating the following **I**, **E** and **M** steps.

- **I step:** *Impute* by using expectation of the missing values $\mathbf{x}^m$ using $\mathbf{x}^o$, θ^{r-1}, t_{ik}^{r-1}.
- **E step:** Compute conditional probabilities $z_i = k|\mathbf{x}_i$ using current value θ^{r-1} of the parameter:

$$t_{ik}^r = t_k^r(\mathbf{x}_i|\theta^{r-1}) = \frac{p_k^{r-1} h(\mathbf{x}_i|\alpha_k^{r-1})}{\sum_{l=1}^K p_l^{r-1} h(\mathbf{x}_i|\alpha_k^{r-1})}. \quad (1)$$

- **M step:** Update ML estimate θ^r using conditional probabilities t_{ik}^r as mixing weights

$$L(\theta|\mathbf{x}_1,\ldots,\mathbf{x}_n,\mathbf{t}^r) = \sum_{i=1}^n \sum_{k=1}^K t_{ik}^r \ln\left[p_k h(\mathbf{x}_i|\alpha_k)\right],$$

- Iterate until convergence

EM Algorithm

Starting from an initial arbitrary parameter θ^0, the mth iteration of the EM algorithm consists of repeating the following **I**, **E** and **M** steps.

- **I step:** *Impute* by using expectation of the missing values $\mathbf{x}^m$ using $\mathbf{x}^o$, θ^{r-1}, t_{ik}^{r-1}.
- **E step:** Compute conditional probabilities $z_i = k|\mathbf{x}_i$ using current value θ^{r-1} of the parameter:

$$t_{ik}^r = t_k^r(\mathbf{x}_i|\theta^{r-1}) = \frac{p_k^{r-1} h(\mathbf{x}_i|\alpha_k^{r-1})}{\sum_{l=1}^K p_l^{r-1} h(\mathbf{x}_i|\alpha_k^{r-1})}. \quad (1)$$

- **M step:** Update ML estimate θ^r using conditional probabilities t_{ik}^r as mixing weights

$$L(\theta|\mathbf{x}_1,\ldots,\mathbf{x}_n,\mathbf{t}^r) = \sum_{i=1}^n \sum_{k=1}^K t_{ik}^r \ln\left[p_k h(\mathbf{x}_i|\alpha_k)\right],$$

- Iterate until convergence

EM Algorithm

Starting from an initial arbitrary parameter θ^0, the mth iteration of the EM algorithm consists of repeating the following **I**, **E** and **M** steps.

- **I step:** *Impute* by using expectation of the missing values $\mathbf{x}^m$ using $\mathbf{x}^o$, θ^{r-1}, t_{ik}^{r-1}.
- **E step:** Compute conditional probabilities $z_i = k|\mathbf{x}_i$ using current value θ^{r-1} of the parameter:

$$t_{ik}^r = t_k^r(\mathbf{x}_i|\theta^{r-1}) = \frac{p_k^{r-1} h(\mathbf{x}_i|\boldsymbol{\alpha}_k^{r-1})}{\sum_{l=1}^K p_l^{r-1} h(\mathbf{x}_i|\boldsymbol{\alpha}_k^{r-1})}. \tag{1}$$

- **M step:** Update ML estimate θ^r using conditional probabilities t_{ik}^r as mixing weights

$$L(\theta|\mathbf{x}_1,\ldots,\mathbf{x}_n,\mathbf{t}^r) = \sum_{i=1}^n \sum_{k=1}^K t_{ik}^r \ln\left[p_k h(\mathbf{x}_i|\boldsymbol{\alpha}_k)\right],$$

- Iterate until convergence

EM Algorithm

Starting from an initial arbitrary parameter θ^0, the mth iteration of the EM algorithm consists of repeating the following **I**, **E** and **M** steps.

- **I step:** *Impute* by using expectation of the missing values $\mathbf{x}^m$ using $\mathbf{x}^o$, θ^{r-1}, t_{ik}^{r-1}.
- **E step:** Compute conditional probabilities $z_i = k|\mathbf{x}_i$ using current value θ^{r-1} of the parameter:

$$t_{ik}^r = t_k^r(\mathbf{x}_i|\theta^{r-1}) = \frac{p_k^{r-1} h(\mathbf{x}_i|\boldsymbol{\alpha}_k^{r-1})}{\sum_{l=1}^K p_l^{r-1} h(\mathbf{x}_i|\boldsymbol{\alpha}_k^{r-1})}. \quad (1)$$

- **M step:** Update ML estimate θ^r using conditional probabilities t_{ik}^r as mixing weights

$$L(\theta|\mathbf{x}_1, \ldots, \mathbf{x}_n, \mathbf{t}^r) = \sum_{i=1}^n \sum_{k=1}^K t_{ik}^r \ln\left[p_k h(\mathbf{x}_i|\boldsymbol{\alpha}_k)\right],$$

- Iterate until convergence

SEM/SemiSEM Algorithms

Drawbacks

- The **I** step may be difficult
- EM algorithm may converges slowly and is slowed down by the imputation step
- Biased estimators

Solution: Use Monte Carlo

- Replace **I** step by a simulation step
- **IS step:** simulate missing values $\mathbf{x}^m$ using $\mathbf{x}^o$, θ^{r-1}, t_{ik}^{r-1}.
- Replace **E** step by a simulation step (Optional)
- **S step:** generate labels $\mathbf{z}^r = \{\mathbf{z}_1^r, ..., \mathbf{z}_n^r\}$ according to the categorical distribution $(t_{ik}^r, 1 \leq k \leq K)$.

SEM and SemiSEM does not converge point wise. It generates a Markov chain.

- $\bar{\theta} = (\theta^r)_{r=1,\dots,R}$
- missing values imputed using empirical MAP value (or expectation)

SEM/SemiSEM Algorithms

Drawbacks

- The **I** step may be difficult
- EM algorithm may converges slowly and is slowed down by the imputation step
- Biased estimators

Solution: Use Monte Carlo

- Replace **I** step by a simulation step
- **IS step:** simulate missing values $\mathbf{x}^m$ using $\mathbf{x}^o$, θ^{r-1}, t_{ik}^{r-1}.
- Replace **E** step by a simulation step (Optional)
- **S step:** generate labels $\mathbf{z}^r = \{\mathbf{z}_1^r, ..., \mathbf{z}_n^r\}$ according to the categorical distribution $(t_{ik}^r, 1 \leq k \leq K)$.

SEM and SemiSEM does not converge point wise. It generates a Markov chain.

- $\bar{\theta} = (\theta^r)_{r=1,...,R}$
- missing values imputed using empirical MAP value (or expectation)

SEM/SemiSEM Algorithms

Drawbacks

- The **I** step may be difficult
- EM algorithm may converges slowly and is slowed down by the imputation step
- Biased estimators

Solution: Use Monte Carlo

- Replace **I** step by a simulation step
- **IS step:** simulate missing values $\mathbf{x}^m$ using $\mathbf{x}^o$, θ^{r-1}, t_{ik}^{r-1}.
- Replace **E** step by a simulation step (Optional)
- **S step:** generate labels $\mathbf{z}^r = \{\mathbf{z}_1^r, ..., \mathbf{z}_n^r\}$ according to the categorical distribution $(t_{ik}^r, 1 \leq k \leq K)$.

SEM and SemiSEM does not converge point wise. It generates a Markov chain.

- $\bar{\theta} = (\theta^r)_{r=1,...,R}$
- missing values imputed using empirical MAP value (or expectation)

SEM/SemiSEM Algorithms

Drawbacks

- The **I** step may be difficult
- EM algorithm may converges slowly and is slowed down by the imputation step
- Biased estimators

Solution: Use Monte Carlo

- Replace **I** step by a simulation step
- **IS step:** simulate missing values $\mathbf{x}^m$ using $\mathbf{x}^o$, θ^{r-1}, t_{ik}^{r-1}.
- Replace **E** step by a simulation step (Optional)
- **S step:** generate labels $\mathbf{z}^r = \{\mathbf{z}_1^r, ..., \mathbf{z}_n^r\}$ according to the categorical distribution $(t_{ik}^r, 1 \leq k \leq K)$.

SEM and SemiSEM does not converge point wise. It generates a Markov chain.

- $\bar{\theta} = (\theta^r)_{r=1,...,R}$
- missing values imputed using empirical MAP value (or expectation)

SEM/SemiSEM Algorithms

Drawbacks

- The **I** step may be difficult
- EM algorithm may converges slowly and is slowed down by the imputation step
- Biased estimators

Solution: Use Monte Carlo

- Replace **I** step by a simulation step
- **IS step:** simulate missing values $\mathbf{x}^m$ using $\mathbf{x}^o$, θ^{r-1}, t_{ik}^{r-1}.
- Replace **E** step by a simulation step (Optional)
- **S step:** generate labels $\mathbf{z}^r = \{\mathbf{z}_1^r, \ldots, \mathbf{z}_n^r\}$ according to the categorical distribution $(t_{ik}^r, 1 \leq k \leq K)$.

SEM and SemiSEM does not converge point wise. It generates a Markov chain.

- $\bar{\theta} = (\theta^r)_{r=1,\ldots,R}$
- missing values imputed using empirical MAP value (or expectation)

SEM/SemiSEM Algorithms

Drawbacks

- The **I** step may be difficult
- EM algorithm may converges slowly and is slowed down by the imputation step
- Biased estimators

Solution: Use Monte Carlo

- Replace **I** step by a simulation step
- **IS step:** simulate missing values $\mathbf{x}^m$ using $\mathbf{x}^o$, θ^{r-1}, t_{ik}^{r-1}.
- Replace **E** step by a simulation step (Optional)
- **S step:** generate labels $\mathbf{z}^r = \{\mathbf{z}_1^r, ..., \mathbf{z}_n^r\}$ according to the categorical distribution $(t_{ik}^r, 1 \leq k \leq K)$.

SEM and SemiSEM does not converge point wise. It generates a Markov chain.

- $\bar{\theta} = (\theta^r)_{r=1,...,R}$
- missing values imputed using empirical MAP value (or expectation)

SEM/SemiSEM Algorithms

Drawbacks

- The **I** step may be difficult
- EM algorithm may converges slowly and is slowed down by the imputation step
- Biased estimators

Solution: Use Monte Carlo

- Replace **I** step by a simulation step
- **IS step:** simulate missing values $\mathbf{x}^m$ using $\mathbf{x}^o$, θ^{r-1}, t_{ik}^{r-1}.
- Replace **E** step by a simulation step (Optional)
- **S step:** generate labels $\mathbf{z}^r = \{\mathbf{z}_1^r, ..., \mathbf{z}_n^r\}$ according to the categorical distribution $(t_{ik}^r, 1 \leq k \leq K)$.

SEM and SemiSEM does not converge point wise. It generates a Markov chain.

- $\bar{\theta} = (\theta^r)_{r=1,...,R}$
- missing values imputed using empirical MAP value (or expectation)

SEM/SemiSEM Algorithms

Drawbacks

- The **I** step may be difficult
- EM algorithm may converges slowly and is slowed down by the imputation step
- Biased estimators

Solution: Use Monte Carlo

- Replace **I** step by a simulation step
- **IS step:** simulate missing values $\mathbf{x}^m$ using $\mathbf{x}^o$, θ^{r-1}, t_{ik}^{r-1}.
- Replace **E** step by a simulation step (Optional)
- **S step:** generate labels $\mathbf{z}^r = \{\mathbf{z}_1^r, ..., \mathbf{z}_n^r\}$ according to the categorical distribution $(t_{ik}^r, 1 \leq k \leq K)$.

SEM and SemiSEM does not converge point wise. It generates a Markov chain.

- $\bar{\theta} = (\theta^r)_{r=1,...,R}$
- missing values imputed using empirical MAP value (or expectation)

SEM/SemiSEM Algorithms

Drawbacks

- The **I** step may be difficult
- EM algorithm may converges slowly and is slowed down by the imputation step
- Biased estimators

Solution: Use Monte Carlo

- Replace **I** step by a simulation step
- **IS step:** simulate missing values $\mathbf{x}^m$ using $\mathbf{x}^o$, θ^{r-1}, t_{ik}^{r-1}.
- Replace **E** step by a simulation step (Optional)
- **S step:** generate labels $\mathbf{z}^r = \{\mathbf{z}_1^r, ..., \mathbf{z}_n^r\}$ according to the categorical distribution $(t_{ik}^r, 1 \leq k \leq K)$.

SEM and SemiSEM does not converge point wise. It generates a Markov chain.

- $\bar{\theta} = (\theta^r)_{r=1,...,R}$
- missing values imputed using empirical MAP value (or expectation)

SEM/SemiSEM Algorithms

Drawbacks

- The **I** step may be difficult
- EM algorithm may converges slowly and is slowed down by the imputation step
- Biased estimators

Solution: Use Monte Carlo

- Replace **I** step by a simulation step
- **IS step:** simulate missing values $\mathbf{x}^m$ using $\mathbf{x}^o$, θ^{r-1}, t_{ik}^{r-1}.
- Replace **E** step by a simulation step (Optional)
- **S step:** generate labels $\mathbf{z}^r = \{\mathbf{z}_1^r, ..., \mathbf{z}_n^r\}$ according to the categorical distribution $(t_{ik}^r, 1 \leq k \leq K)$.

SEM and SemiSEM does not converge point wise. It generates a Markov chain.

- $\bar{\theta} = (\theta^r)_{r=1,...,R}$
- missing values imputed using empirical MAP value (or expectation)

SEM/SemiSEM Algorithms

Drawbacks

- The **I** step may be difficult
- EM algorithm may converges slowly and is slowed down by the imputation step
- Biased estimators

Solution: Use Monte Carlo

- Replace **I** step by a simulation step
- **IS step:** simulate missing values $\mathbf{x}^m$ using $\mathbf{x}^o$, θ^{r-1}, t_{ik}^{r-1}.
- Replace **E** step by a simulation step (Optional)
- **S step:** generate labels $\mathbf{z}^r = \{\mathbf{z}_1^r, \ldots, \mathbf{z}_n^r\}$ according to the categorical distribution $(t_{ik}^r, 1 \leq k \leq K)$.

SEM and SemiSEM does not converge point wise. It generates a Markov chain.

- $\bar{\theta} = (\theta^r)_{r=1,\ldots,R}$
- missing values imputed using empirical MAP value (or expectation)

Mixed Data

Mixed data are handled using conditional independence of the variables.

1. Observation space of the form $\boldsymbol{X} = \boldsymbol{X}_1 \times \boldsymbol{X}_2 \times \ldots \times \boldsymbol{X}_L$
2. $\mathbf{x}_i$ arises from a mixture probability distribution with density

$$f(\mathbf{x}_i = (\mathbf{x}_{1i}, \mathbf{x}_{2i}, \ldots \mathbf{x}_{Li})|\theta) = \sum_{k=1}^{K} \pi_k \prod_{l=1}^{L} h^l(\mathbf{x}_{li}|\boldsymbol{\alpha}_{lk}).$$

3. The density functions (or probability distribution functions) $h^l(.|\boldsymbol{\alpha}_{lk})$ can be any implemented model.

MixAll implements Gaussian, Poisson, Categorical, Gamma distributions.
MixtComp implements Gaussian, Poisson, Categorical and specific distributions for rank and ordinal data.

Sommaire

- Défi Mastodons: Appel à Projet 2016 "Qualité des données"
- Creation of the CloHe (CLustering Of Heterogeneous Data with applications to satellite data records) project
- Members: Mathieu Fauvel (INRA), Stéphane Girard (Inria Grenoble), Vincent vandewalle (Lille2), Crisitan Preda (Université Lille 1)

https://modal.lille.inria.fr/CloHe/

Formosat-2 is no more operational

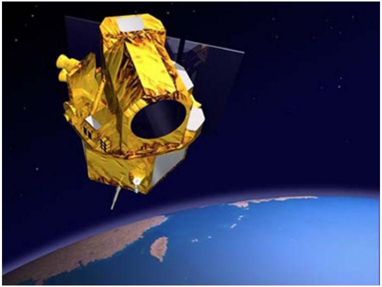


Figure: Formosat-2 furnished multi-spectral data (R, G, B, NIR) with a 8 meter resolution. 17 complete images of France by year

Sentinel-2A start service in 2016.

Figure: Sentinel-2 furnish 13 spectral bandwidths with 4 bandwidths with a 10 meters resolution and 6 bandwidths with a 20 meters resolution. A complete image of France every 5 days

Data Cube

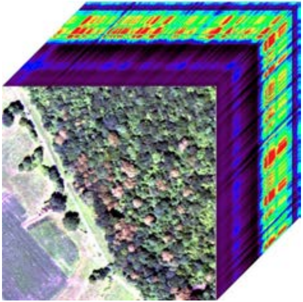


Figure: Sentinel-2 furnish approximately 20TB of images/year, and cover the entire France in 5 days with 1.6 milliard de pixels.

Data Cube

$$\mathbf{X} = (X_{ikt}), \quad i \in I, \quad k \in \{\text{r,v,b,ir}\},$$
$$\mathbf{Y} = (Y_i), \quad i \in J \subset I.$$

with

- $i = (x, y)$ geographic position,
- k spectral band,
- t dates,
- missing values (clouds, ported shadows) at some dates and some positions,
- noisy data (undetected shadows, cloud veil, etc...).
- mixel (mixture of pixel)

Data Cube

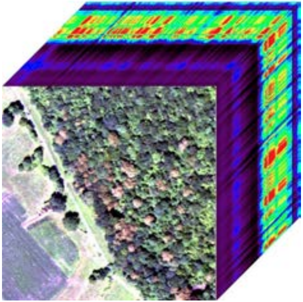


Figure: Sentinel-2 furnish approximately 20TB of images/year, and cover the entire France in 5 days with 1.6 milliard de pixels.

Data Cube

$$
\begin{aligned}
\mathbf{X} &= (X_{ikt}), \quad i \in I, \quad k \in \{\text{r,v,b,ir}\}, \\
\mathbf{Y} &= (Y_i), \quad i \in J \subset I.
\end{aligned}
$$

with

- $i = (x, y)$ geographic position,
- k spectral band,
- t dates,
- missing values (clouds, ported shadows) at some dates and some positions,
- noisy data (undetected shadows, cloud veil, etc...).
- mixel (mixture of pixel)

Data Cube

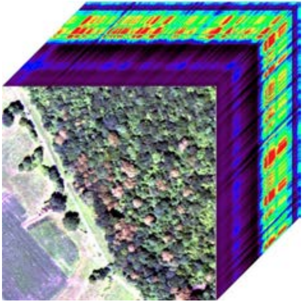


Figure: Sentinel-2 furnish approximately 20TB of images/year, and cover the entire France in 5 days with 1.6 milliard de pixels.

Data Cube

$$\mathbf{X} = (X_{ikt}), \quad i \in I, \quad k \in \{\text{r,v,b,ir}\},$$
$$\mathbf{Y} = (Y_i), \quad i \in J \subset I.$$

with

- $i = (x, y)$ geographic position,
- k spectral band,
- t dates,
- missing values (clouds, ported shadows) at some dates and some positions,
- noisy data (undetected shadows, cloud veil, etc...).
- mixel (mixture of pixel)

Data Cube

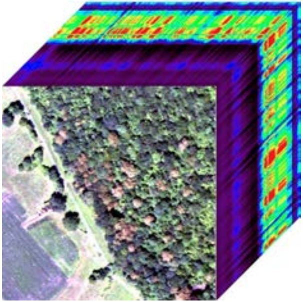


Figure: Sentinel-2 furnish approximately 20TB of images/year, and cover the entire France in 5 days with 1.6 milliard de pixels.

Data Cube

$$\mathbf{X} = (X_{ikt}), \quad i \in I, \quad k \in \{\text{r,v,b,ir}\},$$
$$\mathbf{Y} = (Y_i), \quad i \in J \subset I.$$

with

- $i = (x, y)$ geographic position,
- k spectral band,
- t dates,
- missing values (clouds, ported shadows) at some dates and some positions,
- noisy data (undetected shadows, cloud veil, etc...).
- mixel (mixture of pixel)

Data Cube

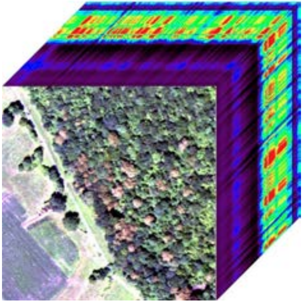


Figure: Sentinel-2 furnish approximately 20TB of images/year, and cover the entire France in 5 days with 1.6 milliard de pixels.

Data Cube

$$\mathbf{X} = (X_{ikt}), \quad i \in I, \quad k \in \{\text{r,v,b,ir}\},$$
$$\mathbf{Y} = (Y_i), \quad i \in J \subset I.$$

with

- $i = (x, y)$ geographic position,
- k spectral band,
- t dates,
- missing values (clouds, ported shadows) at some dates and some positions,
- noisy data (undetected shadows, cloud veil, etc...).
- mixel (mixture of pixel)

Missing data

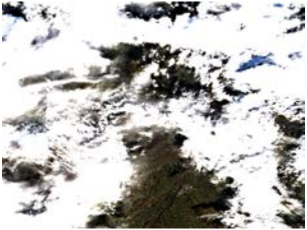


Figure: Very cloudy

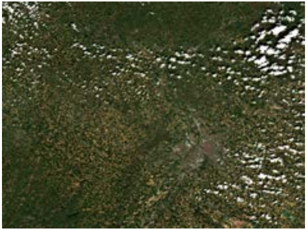


Figure: A few number of clouds

Figure: "sheeps"

Figure: Some clouds with a veil

Noisy Data

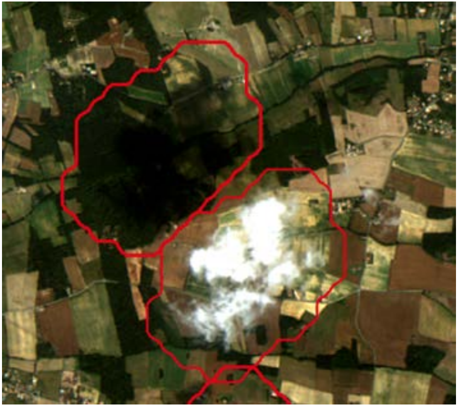


Figure: clouds and their shadows

Non-Uniform sampling

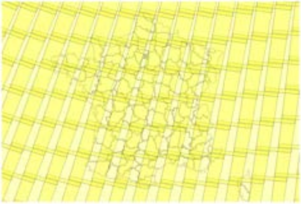


Figure: Path-row grid for Landsat acquisitions. Every path (North-South track) is acquired on the same date every 16 days.

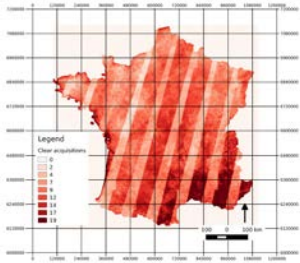


Figure: Map of the number of times that every pixel sees the ground taking into account satellite revisit and cloud cover.

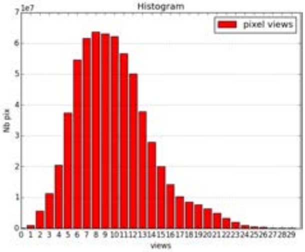


Figure: Histogram of the number of times that every pixel sees the ground taking into account satellite revisit and cloud cover.

Open Access: `http://www.mdpi.com/2072-4292/9/1/95/htm`

Objective

The aim is to be able to cluster the whole France using Sentinel-2 data.

Gaussian modeling

- $Y_i \in \{1, \dots, G\}$,
- $\mathcal{L}(\mathbf{X}_i | Y_i = g) = \mathcal{N}(\boldsymbol{\mu}_g, \Sigma_g)$
- Two kinds of parsimony assumptions on covariance matrices
  - independence between spectra $\Sigma_{g,k}$ of size $T \times T$, ($T = 17$),
  - or independences between times, $\Sigma_{g,t}$ of size $K \times K$, ($K = 4$).
- handle missing values for both models
- Implementations and tests in a R package.

Gaussian modeling

- $Y_i \in \{1, \ldots, G\}$,
- $\mathcal{L}(\mathbf{X}_i | Y_i = g) = \mathcal{N}(\boldsymbol{\mu}_g, \Sigma_g)$
- Two kinds of parsimony assumptions on covariance matrices
  - independence between spectra $\Sigma_{g,k}$ of size $T \times T$, $(T = 17)$,
  - or independences between times, $\Sigma_{g,t}$ of size $K \times K$, $(K = 4)$.
- handle missing values for both models
- Implementations and tests in a R package.

Gaussian modeling

- $Y_i \in \{1, \ldots, G\}$,
- $\mathcal{L}(\mathbf{X}_i | Y_i = g) = \mathcal{N}(\boldsymbol{\mu}_g, \Sigma_g)$
- Two kinds of parsimony assumptions on covariance matrices
  - independence between spectra $\Sigma_{g,k}$ of size $T \times T$, $(T = 17)$,
  - or independences between times, $\Sigma_{g,t}$ of size $K \times K$, $(K = 4)$.
- handle missing values for both models
- Implementations and tests in a R package.

Gaussian modeling

- $Y_i \in \{1, \ldots, G\}$,
- $\mathcal{L}(\mathbf{X}_i | Y_i = g) = \mathcal{N}(\boldsymbol{\mu}_g, \Sigma_g)$
- Two kinds of parsimony assumptions on covariance matrices
  - independence between spectra $\Sigma_{g,k}$ of size $T \times T$, $(T = 17)$,
  - or independences between times, $\Sigma_{g,t}$ of size $K \times K$, $(K = 4)$.
- handle missing values for both models
- Implementations and tests in a R package.

Method

Missing values formation process

Missing At Random (MAR): Probability for a value to be missing does not depends from its value conditionally to the other observations.

Denote $\boldsymbol{x}_{ik}^{+} = \begin{pmatrix} \boldsymbol{x}_i^{\mathrm{O}} \\ \boldsymbol{x}_{ik}^{\mathrm{M}^+} \end{pmatrix}$, $\tilde{\Sigma}_{ik}^{+} = \begin{pmatrix} 0_i^{\mathrm{O}} & 0_i^{\mathrm{OM}} \\ 0_i^{\mathrm{MO}} & \tilde{\Sigma}_{ik}^{\mathrm{M}^+} \end{pmatrix}$ with 0 null matrix, and $\tilde{\Sigma}_{ik}^{\mathrm{M}^+} = \Sigma_{ik}^{\mathrm{M}} - \Sigma_{ik}^{\mathrm{MO}} \left(\Sigma_{ik}^{\mathrm{O}}\right)^{-1} \Sigma_{ik}^{\mathrm{OM}}$. then

$$\Sigma_k^{+} = \frac{1}{n_k^{+}} \sum_{i=1}^{n} \left[(\boldsymbol{x}_{ik}^{+} - \boldsymbol{\mu}_k^{+})(\boldsymbol{x}_{ik}^{+} - \boldsymbol{\mu}_k^{+})' + \tilde{\Sigma}_{ik}^{+} \right]$$

$\tilde{\Sigma}_{ik}^{\mathrm{M}^+}$

is correcting the variance due to the imputation by the mean.

Gaussian modeling

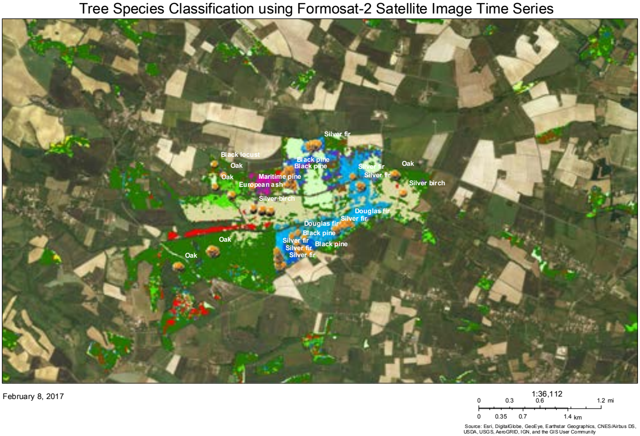


Figure: Tree species classification with $G = 13$

Continuous Model

Main assumption

$$Y|Z = k \sim GP(\mu_k, C_k),\ k = 1, \dots, K \tag{2}$$

where $GP(\mu_k, C_k)$ is a Gaussian Process with mean $\mu_k \in L_2(I)$ and with covariance operator $C_k : I \times I \to \mathbb{R}$.

- mean functions belongs to a J−dimensional subspace

$$\mu_k(t) = \sum_{j=1}^{J} \alpha_{kj}\varphi_j(t),$$

- Covariance function

$$C_k(s,t)(h_k) = \theta_k Q((t-s)/h_k),$$

- Spectrum are independents.

Continuous Model

Main assumption

$$Y|Z = k \sim GP(\mu_k, C_k),\ k = 1, \dots, K \tag{2}$$

where $GP(\mu_k, C_k)$ is a Gaussian Process with mean $\mu_k \in L_2(I)$ and with covariance operator $C_k : I \times I \to \mathbb{R}$.

- mean functions belongs to a $J-$dimensional subspace

$$\mu_k(t) = \sum_{j=1}^{J} \alpha_{kj}\varphi_j(t),$$

- Covariance function

$$C_k(s,t)(h_k) = \theta_k Q((t-s)/h_k),$$

- Spectrum are independents.

Continuous Model

Main assumption

$$Y|Z = k \sim GP(\mu_k, C_k),\ k = 1, \dots, K \tag{2}$$

where $GP(\mu_k, C_k)$ is a Gaussian Process with mean $\mu_k \in L_2(I)$ and with covariance operator $C_k : I \times I \to \mathbb{R}$.

- mean functions belongs to a $J-$dimensional subspace

$$\mu_k(t) = \sum_{j=1}^{J} \alpha_{kj}\varphi_j(t),$$

- Covariance function

$$C_k(s,t)(h_k) = \theta_k Q((t-s)/h_k),$$

- Spectrum are independents.

Continuous Model

Main assumption

$$Y|Z = k \sim GP(\mu_k, C_k),\ k = 1, \dots, K \tag{2}$$

where $GP(\mu_k, C_k)$ is a Gaussian Process with mean $\mu_k \in L_2(I)$ and with covariance operator $C_k : I \times I \to \mathbb{R}$.

- mean functions belongs to a $J-$dimensional subspace

$$\mu_k(t) = \sum_{j=1}^{J} \alpha_{kj}\varphi_j(t),$$

- Covariance function

$$C_k(s,t)(h_k) = \theta_k Q((t-s)/h_k),$$

- Spectrum are independents.

Estimation of Continuous Model

For each i, let $B^i_{\ell,j} = \varphi_j(t^i_\ell)$, $m_{ki} = B^i \alpha_k$ and

$$\Sigma^i_{j,j'}(h_k) = \theta_k Q((t^i_j - t^i_{j'})/h_k) =: \theta_k S^i_{j,j'}(h_k),$$

then

$$y_i | Z_i = k \sim \mathcal{N}_{T_i}(m_{ki}, \theta_k S^i(h_k)),\ k = 1, \dots, K,\ i = 1, \dots, n$$

we end up with K independent minimization problems:

$$(\hat\alpha_k, \hat h_k) = \arg \max_{\alpha_k, h_k, \theta_k} \sum_{Z_i = k} \log \det S^i(h_k) + T_i \log \theta_k + \frac{1}{\theta_k}(y_i - B^i \alpha_k)^\top S^i(h_k)^{-1}(y_i - B^i \alpha_k)$$

Results

About 65% well classified.

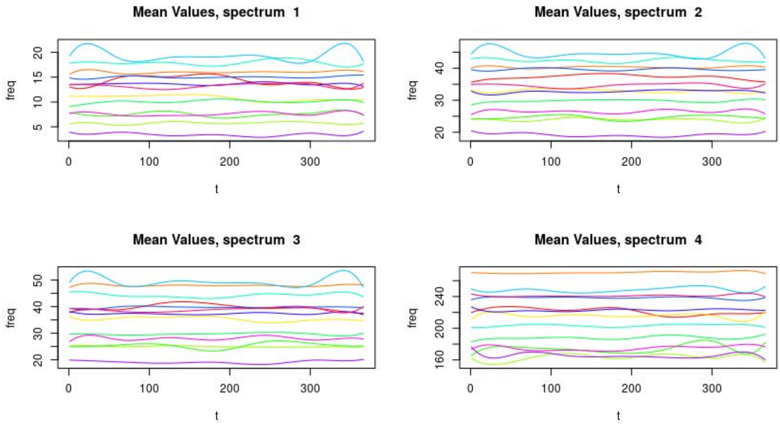


Figure: $G = 13$ spectrum

Mean values

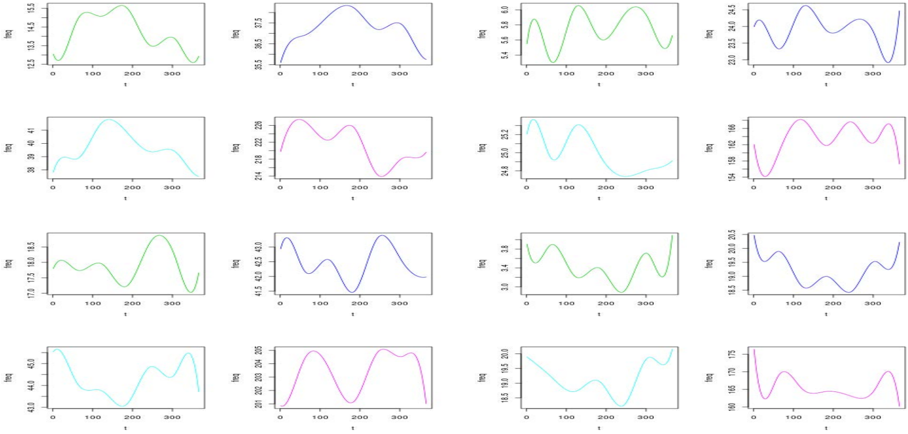


Figure: first, 4th, 7th and 11th classes

Links

- https://cran.r-project.org/web/packages/MixAll/
- https://massiccc.lille.inria.fr/
- https://modal.lille.inria.fr/CloHe/
- http://www.mdpi.com/2072-4292/9/1/95/htm